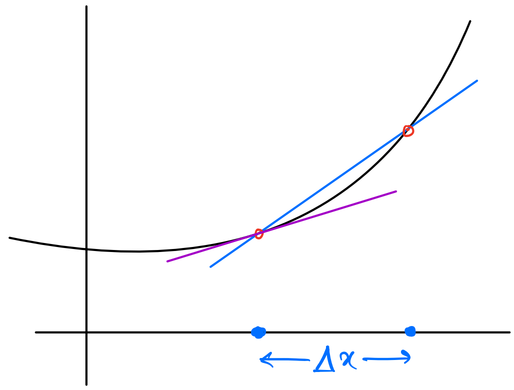
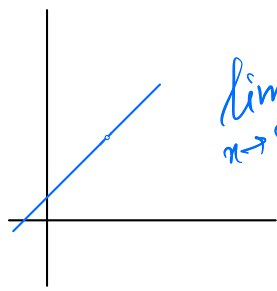
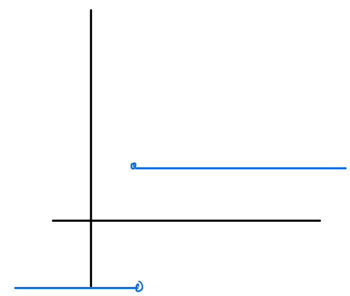


Slope of Tangent Line $= \lim\limits_{\Delta x \to 0}$ slope of secant line.

Eg: $f(x) = \dfrac{x^2-4}{x-2}$, $g(x) = \dfrac{|x-2|}{x-2}$



$\lim\limits_{x\to 2} f(x) = 4$



$\lim\limits_{x\to 2^-} f(x) = -1$

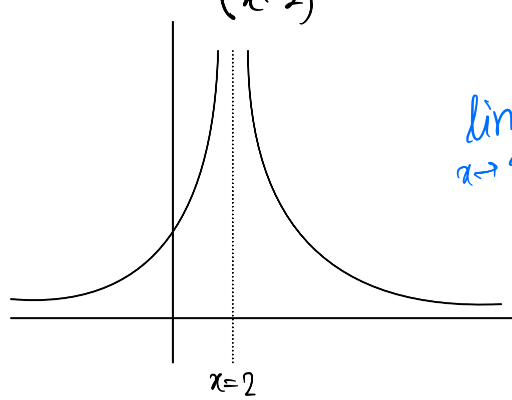
$\lim\limits_{x\to 2^+} f(x) = +1$

One sided

$f(2)$: DNE or undefined

$f(2)$: DNE or undefined.

$h(x) = \dfrac{1}{(x-2)^2}$



$\lim\limits_{x\to 2} f(x) = \infty$

infinite limit

$f(2)$: DNE or undefined.

If $f(x)$ is defined at every other pt except $x=2$.
Then intuitively, if $f(x)$ approach to L as the values of x approaches a

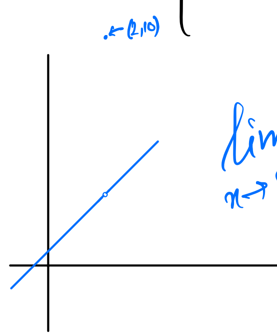
Then we write $\lim\limits_{x\to a} f(x) = L$

Eg. Evaluate $\lim\limits_{x\to 0} \frac{\sin x}{x}$

x	$f(x)$	x	$f(x)$
-0.1	0.9983341	0.1	0.9983341
-0.01	0.99998333	0.01	0.99998333
-0.001	0.99999833	0.001	0.99999833
-0.0001	0.99999999	0.0001	0.99999999

So as we approach near 0, $f(x)$ gradually becoming $0.999999\ldots$ $= 1$.

Eg: $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ 10, & x = 2 \end{cases}$



$\lim\limits_{x\to 2} f(x) = 4$

$f(2)$: Defined <u>but</u> $\lim\limits_{x\to 2}$ exists.

Limit from Left & Right.

Th: $\lim_{x \to a} f(x) \Leftrightarrow \begin{cases} \lim_{x \to a^-} f(x) = L \\ \lim_{x \to a^+} f(x) = L \end{cases}$

Infinite Limits

① from left

② from right

③ from both side.

Eg: $\lim_{x \to 0^-} \frac{1}{x}$

$\lim_{x \to 0^+} \frac{1}{x}$

$\lim_{x \to 0} \frac{1}{x}$

Vertical Asymptote

$$\lim_{x \to a^-} f(x) = +\infty \text{ or } -\infty$$

$$\lim_{x \to a^+} f(x) = +\infty \text{ or } -\infty$$

$$\lim_{x \to a} f(x) = +\infty \text{ or } -\infty$$

If any of the above holds the line $x = a$, is a vertical asymptote of $f(x)$.

Eg. $f(x) = \frac{1}{(x+3)^4}$

$$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^+} f(x) = \lim_{x \to -3} f(x) = \infty$$

Then $x = -3$ is a vertical asymptote.

Th:

(i) $\lim_{x \to a} x = a$

[If $f(x)$ is an algebraic function, then $\lim_{x \to a} f(x) = f(a)$]

(ii) $\lim_{x \to a} c = c$.